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Communicating concepts of academic numeracy through a pattern-based approach

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The term "pattern" is widely used in many domains, either colloquially or in scientific and technical contexts. Regardless of the domain of discourse, however, a pattern articulates fundamental, recurring system structure. As such, it embodies essential design knowledge that can be used in building and maintaining systems. It provides a solution to a problem that balances key forces in a given context. The best patterns are generative, teaching how to build the solution they propose, rather than just explaining it. Patterns are important to the practice of academic numeracy skills advisers for three reasons. First, patterns ground abstract concepts in context to aid problem solving, which is often cited as an important graduate skill and is an integral part of academic numeracy. Second, consideration of context brings with it consideration of constraints or forces that are driven by that context; patterns incorporate consideration of forces into problem solving. Third, success in learning mathematics invariably involves pattern recognition. Successful students recognise that while they may not have seen a given problem before, they know what kind of problem it is, and therefore know how to solve it, or recognise the pattern of the solution. This paper explores the application of a patterns-based approach to teaching academic numeracy skills. We provide an overview of patterns, patterns in pedagogy, and issues specific to the practice of numeracy education. Finally, we detail a key pattern, *Getting Started*, for use by academic numeracy advisers in teaching students problem solving skills, an essential component of academic numeracy. We argue that explicitly articulating underlying patterns such as *Getting Started* helps students recognize, grasp, and apply those patterns, as needed, thus improving their learning.

Key Words: pattern, mathematics, numeracy, communication, problem solving.

1. Introduction to patterns

Although Academic Language and Learning (ALL) advisers are most commonly associated with academic literacy, a number also work, either additionally or solely, as numeracy advisers. Many advisers have no doubt witnessed the emergence of concordancing for use in English language teaching over the last few decades. This so-called "data-driven learning" (Johns & King, 1991, p. iii) promotes a pattern-based, and inductive approach to learning grammar rather than the traditional deductive one based on grammatical rules and definitions. While pattern-based approaches such as concordancing are relatively well known among ALL advisers, the use of patterns to facilitate learning of academic numeracy has not been as clearly detailed for their numeracy colleagues.

The term “pattern” is widely used in many domains, either colloquially, or in technical or in academic contexts. For example, a pattern might be defined in an artistic context as a “regularly repeated arrangement, especially a design made from repeated lines, shapes or colours on a surface” (Cambridge University Press, 2003). In the context of understanding the operation of a business, a pattern might be “a particular way in which something is done, organised or happens” (Cambridge University Press, 2003), or where a physical artefact is being built, “a model or original used as an archetype” (Pickett, 2000). In a software system, a pattern could be a particular recurring structural relationship between classes or object instances (Coad, 1992). Patterns in architecture describe fundamental structural characteristics of buildings and towns (Alexander, Ishikawa, & Silverstein, 1977). In a physical system, a pattern is a configuration that forms as a result of a transformation that breaks symmetry (Stewart & Golubitsky, 1992).

While there are some common features across domains, there is no definitive, universal definition of the pattern concept, perhaps because patterns do not lend themselves to prescriptive definitions; rather, it is consensus about the existence of patterns in a range of existing artefacts that validates them (Winn & Calder, 2002). However, we still need to develop our understanding of patterns at a theoretical as well as practical level in order to identify and use them well. After a detailed study of patterns across many domains, Winn (2006) notes that regardless of the particular domain, a pattern provides a solution to a problem that balances key forces in a given context. A pattern articulates fundamental, recurring system structure and, as such, embodies key design knowledge that can be used in building and maintaining systems. The best patterns are generative, teaching how to build the solution they propose, rather than just explaining it.

A pattern is perhaps most frequently described as a solution to a problem in a context (Coplien, 1996, p. 2; Gamma, Helm, Johnson, & Vlissides, 1993, p. 3; Schmidt, Johnson, & Fayad, 1996). Within a given domain, what may appear to be very different problems often turn out to be the same basic problem occurring in different contexts. A pattern identifies such a recurring problem and a solution, describing them in a particular context to help developers understand how to create an appropriate solution. Patterns thus capture and explicitly state general problem-solving knowledge that is usually implicit and gained only through experience (Buschmann, Meunier, Rohnert, Sommerlad, & Stal, 1996):

> When experts work on a particular problem, it is unusual for them to tackle it by inventing a new solution that is completely distinct from existing ones. They often recall a similar problem they have already solved, and reuse the essence of its solution to solve the new problem. This kind of “expert behavior”, the thinking in problem-solution pairs, is common to many different domains … (p. 2)

Explicitly stating key design knowledge in a pattern can bring that knowledge to the attention of experts who would otherwise be unaware of it. That knowledge can be used to solve what appears to be a new problem with a tried-and-true solution, thus improving the design of new artefacts. But a pattern is more than just a specification for a solution (Coplien, 1996):

> I could tell you how to make a dress by specifying the route of a scissors through a piece of cloth in terms of angles and lengths of cut. Or, I could give you a pattern. Reading the specification, you would have no idea what was being built or if you had built the right thing when you were finished. The pattern foreshadows the product: it is the rule for making the thing, but it is also, in many respects, the thing itself. (p. 3)

A pattern provides some sort of picture of the geometry or shape of the potential artefacts it describes. In a domain like architecture, where Alexander (1977) pioneered the use of patterns in architectural design, geometry has an obvious physical meaning and describes the shape of buildings and other architectural artefacts. Where a mathematics solution is described in terms of a pattern, the pattern must provide insight at a big picture level as to how and why the solution is effective, providing the person using the pattern with more than a rote-learned solution and with the key insight necessary to understanding how to discern when the solution is appropriate and how to apply it in different contexts. In other words, a pattern is both process and product; it articulates both the process required to generate an artefact, and the artefact that will be generated. A pattern therefore does more than just describe the characteristics of a good solution. It also teaches how to create such solutions, and for this reason is often described as *generative* and *structural*:

> The pattern is, in short, at the same time a thing, which happens in the world, and the rule which tells us how to create that thing, and when we must create it. It is both a process and a thing; both a description of a thing which is alive and a description of the process which will generate that thing. (Alexander, 1979, p. 247)
>
> Once we understand buildings in terms of their patterns, we have a way of looking at them which makes all buildings, all parts of a town similar … We have a way of understanding the generative processes which give rise to these patterns. (Alexander, 1979, p. 11)

Alexander (1979, p. 84) notes, however, that although patterns "… seem like elementary building blocks, [they] keep varying and are different every time they occur". Patterns articulate recurring structure, but that structure changes according to context. Alexander argues that the invariant structure that a pattern articulates has to do with relationship, rather than being embodied in what would traditionally be seen as an entity. It is the structural relationships that remain constant across a variety of contexts. For example, while each gothic cathedral has a nave and aisles, the particulars of nave and aisle are quite different from one cathedral to another. Neither nave nor aisle by itself forms a pattern. The pattern is the structure generated by the invariant relationship between nave and aisle in gothic cathedrals: "within a gothic cathedral … the nave is flanked on both sides by parallel aisles" (Alexander, 1979, p. 90).

While a pattern does generate structure, it does more than that; a pattern generates a structure or solution that balances key forces. Developing a good solution requires understanding both the forces that bring about the problem and how those forces interact. For example, some researchers (Nunes, Schliemann, & Carraher, 1993, pp. 147-154) argue that for mathematics teaching to be effective, it must take into account forces such as the maths anxiety felt by some students. If such forces are ignored and dismissed as irrelevant, the mathematical teaching, even if theoretically competent, may be of little use because it is not provided in a way that takes into account all the key factors affecting the learning environment. Developing a solution to a problem therefore requires balancing possibly conflicting forces in such a way that the solution structures developed are stable enough to be effective. The fact that a pattern encompasses problem, solution and context enables it to articulate the centrality of forces to problem solving, and to provide stable solutions to problems (Weiss, 2003):

> The documentation of a pattern goes beyond documenting a problem and its solution. It also describes the forces or design constraints that give rise to the proposed solution. These are the undocumented and generally misunderstood features of a design. Forces can be thought of as pushing or pulling the design towards different solutions. A good pattern balances the forces. (p. 712)

In domains such as architecture and computer science, which have been at the forefront of patterns research, patterns are usually documented according to one of several typical forms. In this paper, we use an amalgam of existing forms (Coplien, 1996) including the following sections: *Name*, *Problem*, *Context*, *Forces*, *Solution*, *Rationale*, *Example*, *Resulting Context*. The *Name* section is not explicitly labelled as such, but is a key word or phrase that describes the pattern. *Problem* describes the problem or difficulty the pattern addresses and *Context* specifies when to apply the pattern, as well as the things that, if changed, would invalidate the pattern. A *Forces* section is important because patterns are not just rules to be followed blindly, but involve balancing forces or constraints that define the key trade-offs in the development of a solution to a problem. The *Solution* section addresses the problem stated in the problem section and should be detailed enough to be useful but general enough to address a broad context. *Rationale* highlights why the solution is necessary, and *Example* describes the general concept that is the focus of the pattern in a specific context. *Resulting Context* wraps up the pattern by noting which forces have been resolved and which new problems may arise as a result of this pattern.

The remainder of this paper provides first a theoretical and then a more practical discussion of the use of patterns to facilitate the learning of numeracy skills. Section 2 outlines the application of patterns to education and Section 3 discusses issues specific to patterns and numeracy. Section 4 outlines and analyses a key pattern, *Getting Started*, for use by academic numeracy advisers to facilitate the learning of problem solving skills, an essential component of academic numeracy. Section 5 wraps up the paper with conclusions and suggestions for future work.

2. Patterns in education

Kuhn (1962, pp. 10-22) describes a paradigm as a thought pattern that defines the set of practices of a scientific community at a given point in time. He points out that science is not limited to the observations, laws, and theories taught in textbooks but that often implicit paradigms underlie the way laws and theories have been developed, and understanding of the underlying paradigm is critical for the science student. Floyd (1979) applies Kuhn's insight to the teaching of computer science, discussing the teaching of computer programming using programming languages such as Pascal and FORTRAN:

> If I ask another professor what he teaches in the introductory programming course, whether he answers proudly "Pascal" or diffidently "FORTRAN," I know that he is teaching a grammar, a set of semantic rules, and some finished algorithms, leaving the students to discover, on their own, some process of design.
>
> …
>
> [You should] identify the paradigms [patterns] you use, as fully as you can, then teach them explicitly. They will serve your students when Fortran has replaced Latin and Sanskrit as the archetypal dead language. (pp. 458-459)

Floyd's point is that the same fundamental patterns underlie programming tasks, regardless of the particular programming language used. Similarly, common patterns underlie the ability to communicate, regardless of the particular spoken language used. Teaching is made much more effective when those patterns are explicitly recognised and taught alongside the syntax for a particular language.

Devlin (1998) points out that in recent decades, mathematics has come to be characterised as the science of patterns. He highlights a number of entities and processes – from snowflakes to wallpaper, from motion and change to the Golden Ratio – and describes how the mathematics underlying those phenomena might be described in terms of patterns. Such characterisations of mathematics as the science of patterns should not be surprising given Polya's (1962; 1965) seminal work on mathematics and problem solving, and discussion of related patterns. Rather than subscribing to a narrow definition of problem solving as being able obtain the correct answer to a particular problem or type of problem by applying a standard, learned technique, Polya argues that the skill of problem solving is grounded in the ability to be a good guesser and apply heuristic strategies to independently solve challenging problems. Problem solving is then much more related to guessing, insight, and discovery than it is to formalistic deductive techniques. Based on this point of view, Polya conceptualises mathematics as problem solving, and argues that the teaching of mathematics ought to focus on "teaching to think" (1965, p. 100) rather than merely imparting information. He thus argues that an exploratory, discovery phase ought to precede the teaching of formal concepts, enabling students to experience active rather than passive learning.

Polya (1962) links his approach to solving problems using heuristics to the concept of pattern:

> Solving problems is a practical art, like swimming, or skiing, or playing the piano: you can learn it only by imitation and practice.
>
> …

> If you wish to derive the most profit from your effort, look out for such features of the problem at hand as may be useful in handling the problems to come. A solution that you have obtained by your own effort or one that you have read or heard, but have followed with real interest and insight, may become a *pattern* for you, a model that you can imitate with advantage in solving similar problems. (p. v)

In the first part of his book, Polya (1962) aims to familiarise learners with useful patterns for doing mathematics. He characterises various mathematical problems in terms of patterns, highlighting the importance of awareness and understanding of the underlying pattern of a solution to the ability to recognise and solve other problems that can be characterised in terms of the same underlying pattern. In describing each pattern, Polya first begins with an example, then generalises from that example to obtain a pattern, and finally describes the pattern in the context of several other examples.

Schoenfeld (1992) builds on Polya's and others' work in his writing on learning to think mathematically. He argues that teaching mathematics effectively requires incorporating both epistemology and ontology, because it is the cultural framework within which we live and work that shapes the kind of approach we take to problem solving. Almost all of western education sits within an epistemological framework which defines knowledge as "the sum total of what we know" (Schoenfeld, 1992, p. 25). The focus of mathematics education has thus been primarily on content, facts, and procedures; mathematics knowledge is understood as content-based rather than being a capacity to think independently. Schoenfeld (1992) points out, however, that over the past two decades there has been a significant change in the way which mathematics is conceptualised:

> … Traditionally one defines what students ought to know in terms of chunks of subject matter, and characterizes what a student knows in terms of the amount of content that has been "mastered". As natural and innocuous as this view of "knowledge as substance" may seem, it has serious entailments …
>
> Over the past two decades there has been a significant change in the face of mathematics … and in the community's understanding of what it is to know and do mathematics … The main thrust of this reconceptualization is to think of mathematics, broadly, as "the science of patterns". (pp. 25-28)

This change in conceptualisation represents a shift in focus from content to process; from rules and procedures to a way of thinking. It represents a growing understanding that mathematics is more akin to other scientific disciplines in the need to gather data and test theories using heuristics. And it represents a growing understanding that doing mathematics is a social and collaborative act.

3. Patterns and numeracy

Kemp (2005, pp. 28-36) points out that the term numeracy is defined and used in the literature diversely, from referring to a collection of basic mathematical skills through to a much richer understanding of mathematics and its application in a variety of contexts. Given the increasing emphasis on contextualising mathematics teaching, however, we use the following definition (AAMT, 1997) :

> Numeracy involves using mathematics to achieve some purpose in a particular context.
>
> …
>
> To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life. In school education, numeracy is a fundamental component of learning, performance discourse and critique across all areas of the curriculum. It involves the disposition to use, in context, a combination of:

> underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical, and algebraic); mathematical thinking and strategies; general thinking skills and a grounded appreciation of context. (p. 15)

Steen (1998) notes that the importance of being numerate is increasing in society as we increasingly rely on computers and related data. Kemp (2005, p. 4) emphasises this point, highlighting the wide variety of everyday activities requiring basic numeracy, including interpreting political polls, understanding the implications of giving medical consent, and understanding safety regulations. When it comes to developing numeracy for use in everyday life, Kemp (2005, pp. 41-79) argues that the two main factors that contribute to numerate behaviour are the way that students learn mathematics and how to apply it to real world situations, and the development and use of cognitive skills. We argue that the use of patterns is critical for developing numeracy because they facilitate the learning of mathematics in appropriate ways and the development of appropriate cognitive skills. In particular, we argue that patterns are important to academic numeracy for three main reasons, as outlined below.

First, patterns integrate knowledge and its application; they ground what would otherwise be abstract concepts in context. Each pattern incorporates a recurring solution, together with an explanation of the contexts in which that solution applies and an example of the pattern described in at least one context. Brown, Collins, and Duguid (1989) argue that social and physical context are beginning to be recognised as critical to the development of usable, robust knowledge:

> Recent investigations of learning … challenge this separating of what is learned from how it is learned and used. The activity in which knowledge is developed and deployed, it is now argued, is not separable from or ancillary to learning and cognition. Nor is it neutral. Rather, it is an integral part of what is learned … activity and situations are integral to cognition and learning … different ideas of what is appropriate learning activity produce very different results. We suggest that by ignoring the situated nature of cognition, education defeats its own goal of providing usable, robust knowledge. And conversely, we argue that approaches such as cognitive apprenticeship (Collins, Brown & Newman, 1989) that embed learning in activity and make deliberate use of the social and physical context are more in line with the understanding of learning and cognition that is emerging from the research. (p. 32)

Kemp (2005, pp. 44-52) notes that there is increasing and widespread recognition that the teaching of numeracy in a way that grounds general mathematical skills in relevant application contexts is critical to students’ overall mastery of numeracy concepts. Lave (1985; 1988) emphasises this, noting that people who cope easily with the mathematics required in familiar, practical contexts, often struggle with similar mathematics when it is not contextualised. Lester (1989) argues that embedding mathematical problems in meaningful context is critical for motivating and sustaining problem-solving activity, and Boaler (1998) points out that at least one reason that students do not use mathematical methods learned in school when outside of school is that they do not understand those methods well enough to know how to apply them in an out-of-school context. In contrast, learning mathematics in an open, project-based environment appears to improve students’ ability to apply mathematics skills in different contexts. In a later study, Boaler (2000) further notes that students reported that:

> … they did not even attempt to make use of school-learned methods in the real world, not because of the form or structure of the problems they encountered … but because the environments of the classroom and their everyday lives were too disparate. The students believed that adopting classroom practices in the real world was inappropriate, so they did not attempt to draw on school mathematics. (p. 114)

The second reason that patterns are important for teaching numeracy is that patterns explicitly articulate and are defined by key forces that shape the contextual solution to a problem. Each description of a pattern includes a list of key forces that make the pattern necessary, and the solution of a pattern is a structure that balances those forces. Without the awareness that knowledge of the pattern brings, these forces are often not well understood, even though they are critical to the development of an effective solution (Coplien, 1996, pp. 9-10, pp. 9-10).

Using mathematics in context means using it in the presence of constraints or forces that shape the problems being solved and influence the mathematics used to solve the problem. Zevenbergen (1997) argues that mathematics education needs to take account of how people work in the real world, and points out that experts often use informal methods of calculation based on observed context and constraints that are at least as accurate and more efficient than formal measuring methods. For example, when building a pool, workers construct a frame according to building regulations without doing any formal measuring, but by building the frame and judging appropriate angles and distance by estimation. Boaler and Greeno (2000) make the case that to provide students with the best opportunity to see the relevance of what they are learning to the real world, mathematics must be taught in a classroom where the culture is one of social interaction, and realistic constraints, negotiation, and learning all contribute to problem solving. In contrast, where mathematics is seen to only involve artificial real-world problems, students are much more likely to struggle to apply their mathematical learning to the real world.

The third reason why patterns are important to numeracy education is that pattern recognition is key to problem solving in mathematics. Masilinga, Davidenko, and Prus-Wisnioska (1996, p. 177) point out that in order to be numerate, students need to be able to develop “generalisable schemas” that can be applied in a variety of contexts. If students only understand particular problem-solving methods in one specific context, they are unlikely to be able to transfer their problem-solving knowledge to other, appropriate contexts. Students need tools that help them make the connection between the “generalisable schema”, the contexts in which it can be applied, and the reasons why it is appropriate to apply a particular schema in a particular context. Adey and Shayer (1994, pp. 71-73) discuss the importance of “bridging” or “reflection” to problem solving in mathematics. They developed a teaching methodology consisting of five central categories, one of which is bridging. Bridging involves relating new concepts to other examples in science, mathematics, or everyday life, and encourages students to recognise underlying similar problem-solving techniques applying in different contexts. The ability to do this kind of pattern recognition is, according to Adey and Shayer (1994), critical to the ability to apply learned theoretical knowledge appropriately in different situations, and the ability to do the pattern recognition is facilitated by explicitly articulating the patterns (1994):

> If bridging is the conscious transfer of a reasoning pattern from a context in which it is first encountered to a new context, then the transfer is most likely to be effective if the reasoning pattern has been made conscious and verbalised. (p. 73)

4. Applying patterns in numeracy education

As numeracy advisors, we have enhanced our teaching by making explicit underlying patterns whose recognition facilitates learning. In Section 4.1, we outline such a pattern using the form described in Section 1. In Section 4.2, we discuss the relationship between that pattern and the theory discussed in earlier sections.

4.1. An example pattern

For many students still to develop academic numeracy skills, reading a mathematics problem does not give them any idea of where to start when it comes to solving that problem. Yet, many documented problem-solving strategies can help with this task. The following pattern, *Getting Started*, addresses the issue of how to begin the process of solving a mathematics problem.

Getting Started

Context: you've read a mathematics problem and feel overwhelmed or confused.

***Problem*:** how to get started working on the mathematics problem.

***Forces*:**

- Reading the maths problem might make you anxious or stressed, making it harder for you to focus on solving the problem (Rogoff & Lave, 1984, pp. 212-219).
- If you can't see the answer straight away you might assume you will never be able to solve the problem (Boaler, 1999).
- Focus is by nature local; you will only be able to focus on one part of a complex problem at any one time (Raskin, 2000, pp. 9-32).
- A similar approach can be used to solve a collection of mathematical problems of the same type (Polya, 1962).
- If you are able to see the problem in a familiar context, it may help to motivate your learning (Lester, 1989).

***Solution*:** Try to express, in everyday language, what the problem asks for. Once you can do this, think about how you would find the answer to what is asked for. Try to express how to find that answer in "maths language"; in other words, translate your everyday language answer into mathematical symbols, as appropriate (Klinger, 2006). You may find it helps to read the question from left to right, top to bottom, writing down word-by-word or phrase-by-phrase in maths language what the words tell you. If you need to, you can break sentences down into very small, manageable fragments. At any point, you may like to draw a picture of the problem to improve your understanding.

***Rationale*:** Many people find it hard to draw out the necessary overall information from a textual description of a mathematics problem. This can be for a number of reasons, but key issues include: being overwhelmed by the problem and unable to find a focus point of "where to start"; a lack of belief that if a solution path is not immediately obvious one can still be worked out; being unable to recognise the "pattern" of the problem – that is, recognise what kind of problem is being presented and therefore what kind of solution might be appropriate; and a lack of understanding as to why solving the problem is relevant or important. This solution addresses these key issues in a number of ways. By providing a starting point – what is asked for – the solution gives the problem solver a place to start, and having a place to start can help reduce anxiety. The solution provides a problem-solving strategy broken down into small steps, so that even if the problem solver cannot see how to solve the problem immediately, they have a guide to follow in terms of a process that can lead them to a solution (Paulos, 1988, pp. 87-89). This can also reduce anxiety and enables the problem solver to focus on one thing at a time as they seek to solve a complex problem.

Examples:

Mathematics Problem 1: If the cost of a main course at a restaurant is listed as $26.90 but you know the total cost of the main course will include an additional 10% GST, what is that total cost?

In this problem, what is asked for is the total cost of the food. The question says that the total cost of the food includes "an additional" 10% GST. The words "an additional" tell you that the total cost includes the $26.90 on the menu plus something more – the 10% GST. So, you could write:

Total Cost = $26.90 + 10% GST

The next step is to work out the 10% GST. To calculate 10% of something, you need to know that 10% is the same as one-tenth and can be written as 1/10, or 10/100, or 0.1. You also need to know that calculating a percentage involves multiplication; to get 10% of $26.95 you do the following calculation:

$$\$26.95 \times \frac{10}{100} = \$2.69 \text{ (The } \frac{10}{100} \text{ is used because it is corresponds to 10\%.)}$$

So, now you can say:

Total Cost = \$26.90 + \$2.69 = \$29.59

You have obtained what was asked for and solved the problem.

Mathematics Problem 2: You are asked to bake a cake using 2 cups of flour, 1 cup of sugar, and ½ a teaspoon of vanilla, along with some other ingredients. If you want to triple the recipe, how much vanilla, sugar, and flour will you need?

In this problem, what is asked for is how much vanilla, sugar, and flour you will need if you triple the recipe. Tripling a recipe means making three times as much, so each ingredient is needed in three times the amount that was in the original recipe. So, there would be:

$2 \times 3 = 6$ cups of flour

$1 \times 3 = 3$ cups of sugar

$\frac{1}{2} \times 3 = 1\frac{1}{2}$ teaspoons of vanilla

You have obtained what is asked for and solved the problem.

Mathematics Problem 3 (Oldham Sixth Form College, 2005a)*:* A salmon can swim at 12 m/s with the current and at 8 m/s against it. Find the speed of the current and the speed of the salmon in still water. Let *c* represent the speed of the current and *s* represent the speed of the salmon in still water.

In this problem, what is asked for is the speed of the current and the speed of the salmon in still water. Having identified what is asked for, if you are still having trouble seeing how to get a solution, try reading the problem from left to right, top to bottom, and translating it into maths language. The problem starts by saying that "a salmon can swim at 12m/s with the current". If the salmon is swimming with the current, then its speed is coming from both the current and the salmon's own speed (its speed in still water). So, the salmon's speed with the current can be written as its speed in still water plus the speed of the current, as follows:

s + c = 12

(where *s* is a symbol for the speed of the salmon in still water, *c* is a symbol for the speed of the current, and 12 is the speed of the salmon when swimming with the current).

The problem also says that the salmon can swim at 8 m/s against the current. When the salmon is swimming against the current, its speed is its own speed in still water less whatever speed it loses by fighting against the current. This can be written in maths language as follows:

s - c = 8

(where *s* is a symbol for the speed of the salmon in still water, *c* is a symbol for the speed of the current, and 8 is the speed of the salmon when swimming against the current).

Once you have these two equations, the problem can be solved easily using simultaneous equations, but since this pattern focuses on getting started, we leave the discussion on simultaneous equations for another place.

Mathematics Problem 4 (Oldham Sixth Form College, 2005b)*:* Two crude oil feed streams are blended to make up a single feed to a distillation column. For simplicity we will assume that while in the distillation column, the crude oil is split into 3 components: Liquefied Petroleum Gas (LPG), Light Virgin Naphtha (LVN) and Petrol. Each crude oil input is composed of a different percentage of each product. Using the data in Table 1, find the percentage of LPG in each of the two input feed crudes. Hint: you will need two equations.

Table 1. Crude oil input values and corresponding output values for LPG, LVN, and petrol, for two different cases.

	Crude Oil Input (m^3/hr)		Total Crude Oil Input (m^3/hr)	Outputs (m^3/hr)			Total Outputs (m^3/hr)
	Stream A	Stream B		LPG	LVN	Petrol	
Case 1	100	100	200	25	45	130	200
Case 2	130	150	280	34.5	62.5	183	280

In this problem, what is asked for is the percentage (or equivalently the fraction) of LPG in each of the two (input) feed crudes. Having identified what is asked for, if you still cannot see where to start, you may find that drawing a picture helps. The problem text says that "two crude oil feed streams are blended to make up a single feed to a distillation column. For simplicity we will assume that while in the distillation column, the crude oil is split into 3 components …". This could be drawn as detailed in Figure 1.

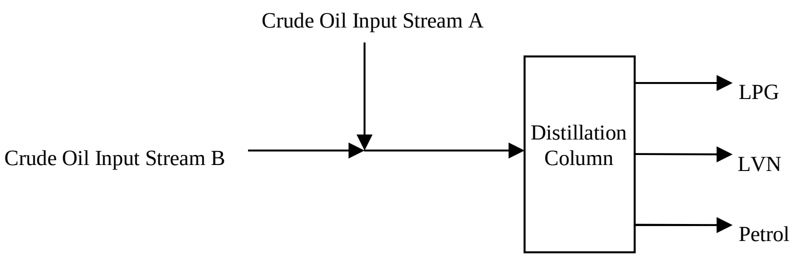

Figure 1. Crude oil input to a distillation column from two different feeds, and output to three different components.

In Table 1, the problem provides information about the percentages of LPG, LVN, and petrol that are distilled from two different compositions of crude oil. The problem asks you to focus on LPG. In the first case, 100 m^3/hr is inputted from both streams A and B and this results in 25 m^3/hr of LPG. Letting *a* be the fraction of LPG in stream A and *b* the fraction of LPG in stream B, this can be written in maths language as follows:

$100a + 100b = 25$ m^3/hr of LPG.

In the second case, 130 m^3/hr is inputted from stream A and 150 m^3/hr is inputted from stream B, resulting in 34.5 m^3/hr of LPG. This can be written in maths language as follows:

$130a + 150b = 34.5$ m^3/hr of LPG.

Once you have these two equations, the problem can be solved easily using simultaneous equations, but since this pattern focuses on getting started, we leave the discussion on simultaneous equations for another place.

***Resulting Context*:** The student has a place to start, and a question they can answer in everyday language as a place to start, rather than diving straight away into mathematical symbols. The student has some guidelines to further proceed with solving the problem, such as translating text into mathematical symbols.

4.2. Discussion

In Section 1, we noted that the best patterns are generative, teaching how to build the solution they propose, rather than just describing it. We argue that *Getting Started* is generative, largely because it focuses on process, rather than content, which Polya (1962, 1965) notes is critical to teaching students how to think in a way that allows them not just to solve the problem at hand, but to be able to generate solutions to other problems by recognising the pattern of a solution. For example, a student who knows that a good place to start is “What is being asked for?” is in a better place to solve new problems than a student who has rote-learned the content of a solution for one problem without understanding how that solution works or why it is applicable.

Another point we made in Section 1 was that patterns provide “big picture” understanding of the problems they address. *Getting Started* provides a student with big picture understanding in the sense that it makes the student more aware of their particular needs with respect to numeracy skills development. Instead of just thinking, for example, “I’m bad at maths”, the student can realise that they are not necessarily bad at maths, but in fact simply need help, say, translating everyday language to maths language, or with a general process that they can apply when a solution is not obvious.

In Section 2, we noted that awareness and understanding of key, underlying patterns can facilitate learning. We argue that *Getting Started* is such a pattern because of its focus on the learning of problem solving skills and significant research evidence, as discussed in Sections 2 and 3, of the importance of problem-solving skills for academic numeracy.

In Section 3, we highlighted three reasons why patterns are important to academic numeracy: for integration of knowledge and its application, for articulation of key forces and context, and because pattern recognition is key to doing mathematics. The main way in which *Getting Started* integrates knowledge and its application is through the examples that form part of the pattern. The pattern’s solution which is first described in general terms is then illustrated with several examples from different contexts. The pattern explicitly lists key forces that contribute to defining the problem, and the “Rationale” section provides discussion of how the solution addresses those forces. Articulating *Getting Started* in pattern form, with the headings such as “Context”, “Problem”, and “Solution”, is not essential, but does help to make the underlying pattern explicit, increasing the student’s awareness of the pattern and also highlighting important aspects of the pattern. For example, by explicitly naming “Context”, the student is made aware that the pattern has a context, and is not universally applicable, and is also provided with information about what the context of the pattern is, facilitating future, effective use of the pattern in other situations.

5. Conclusions

Patterns have always been used by intuitive mathematicians to solve problems. Explicitly articulating those patterns allows for them to be used by less confident mathematicians when developing problem solving skills. Of particular value for those teaching numeracy is that patterns explicitly incorporate context and constraints into problem solving, as well as identifying key types of reasoning used repeatedly to solve mathematical problems. Patterns name and develop explanations, principles and analogies and explore the contexts to which they apply. Patterns thus fit well with recent thinking (Kemp, 2005, pp. 27-95; Schoenfeld, 1992) that emphasises teaching in context and the development of higher level cognitive skills as means of improving numeracy skills.

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